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Your Name Goes On This Line

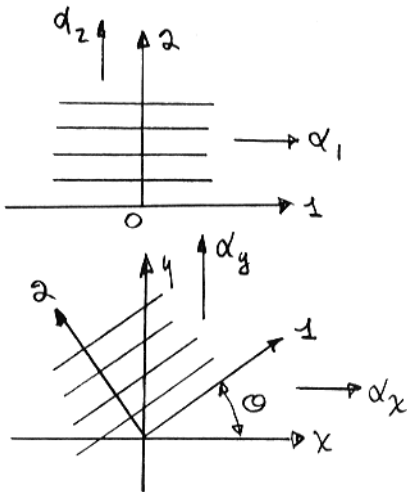
**E98 Final Exam**  
**14 December 1998**

In this examination you may use the class texts and any notes that you have taken in class or made in preparation for the exam. You may also use your homework and our posted homework solutions. All other references are forbidden. There are six problems worth 33 points each. Your name is worth 2 points. The problems are of equal weight but *not* of equal difficulty. There is partial credit. Please write neatly and *on one side* of the paper only. If you use any additional paper, please attach it immediately following the sheet on which the problem statement appears.

1. The strain-temperature relation for planar deformation of an orthotropic plate takes the form:

$$\begin{Bmatrix} \varepsilon_{11} \\ \varepsilon_{22} \\ \gamma_{12} \end{Bmatrix} = \begin{Bmatrix} \alpha_1 \\ \alpha_2 \\ 0 \end{Bmatrix} (T - T_{\text{ref}})$$

for stress-free temperature effects related to the reference axes. Consider now the rotated set of axes as shown. Find the expression for  $\varepsilon_{xx}$ , i.e., the normal strain associated with the rotated  $x$ -axis, in terms of the reference thermal expansion coefficients  $\alpha_1$ ,  $\alpha_2$ , the temperature change  $T - T_{\text{ref}}$ , and the angle  $\theta$ . In particular, find  $\alpha_x(\alpha_1, \alpha_2, \theta)$  ( $\alpha_x$  as a function of  $\alpha_1, \alpha_2, \theta$ ), where  $\varepsilon_{xx} = \alpha_x \cdot (T - T_{\text{ref}})$ .



2. A through-thickness center cracked plate of a medium-carbon steel alloy has dimensions  $b = 40 \text{ mm}$  ,  $t = 15 \text{ mm}$  . For a safety factor of three against brittle fracture, what is the maximum permissible load on the plate if the crack half-length,  $a$  , is:
- 10 mm and,
  - 24 mm ?

For sufficiently long specimens, i.e.,  $h/b > 1.5$  , the dimensionless factor,  $Y$  can be calculated as

$$Y = \frac{(1 - 0.5\alpha + 0.326\alpha^2)}{\sqrt{1 - \alpha}}$$

where  $\alpha = a/b$  .

3. Suppose that fracture occurs in a single crystal of MgO when a critical tensile stress,  $\sigma_f = 30,000 \text{ psi}$ , is resolved across  $\{100\}$  planes, and that yielding occurs when a critical resolved shear stress,  $\tau_{\text{crss}} = 20,000 \text{ psi}$ , is set up along the  $\langle 110 \rangle$  slip directions lying in  $\{110\}$  planes. Will the crystal deform plastically before fracturing when a tensile stress is applied along the  $[100]$  direction?

4. Suppose that the compounds BeO, MgO, and CaO could be constructed by placing the oxygen ions in an FCC array and by putting the cations into the octahedral sites. (The ionic radii for the ions in question are found on the inside front cover)
- If the oxygen ions are arranged in a close-packed array (FCC), which of the cations, Be, Mg, or Ca, would most nearly fit into the octahedral hole provided by the oxygen ions?
  - Which of the cations would cause the most distortion in the oxygen lattice?
  - Calculate the radius of a cation that would fit exactly into the octahedral hole.

5. Consider a leaf spring, supporting a point load,  $F$ , as shown in Figure 13.28. The deflection at the center,  $\delta$ , is given by

$$\delta = \frac{FL^3}{4Ebd^3} .$$

The maximum stress (which occurs at the bottom surface) is given by

$$\sigma = \frac{3FL}{2bd^2} .$$

In a spring, one wants to maximize the deflection that the spring can undergo before permanent (or plastic) deformation occurs. Assuming  $d$  and  $L$  are fixed by design constraints, determine which material from the following list would make the best spring.

TF00 temper beryllium-copper

carbon fiber-epoxy composite  $V_f = 0.6$

Polycarbonate (PC)

6. You have been tasked with designing a flat-plate capacitor. It must have a capacitance of  $1 \mu\text{F}$  at an operating voltage of  $100 \text{ V}$ . Your choice of materials is either soda-lime glass or polyethylene (See Table 19.4).
  - a. Which material will make the lightest capacitor?
  - b. Give the dimensions of the dielectric in the finished capacitor.